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GLOBAL STABILITY ANALYSIS OF MICROWAVE CIRCUITS BY A FREQUENCY-DOMAIN APPROACH

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ABSTRACT

The paper presents a novel computer algorithm allowing a global stability analysis of any nonlinear microwave circuit to be carried out in a general-purpose CAD environment. This is obtained through a systematic application of bifurcation theory in a way compatible with the frequency-domain description of the linear subnetwork.

INTRODUCTION

A global stability analysis should complement the design of any active or nonlinear microwave circuit, since it is the only way to provide and in-depth knowledge of circuit behavior not to be confined to a small neighborhood of the nominal operating point. This kind of analysis can be produced in a systematic way making use of the principles of bifurcation theory {e.g., 1, 2}. For a parametrized system, solution paths in the state space bifurcate at those parameter values for which system stability undergoes an abrupt qualitative change, that is, the real part of one (at least) natural frequency changes sign. The topological and the stability-exchange properties of bifurcations have been studied extensively in the mathematical literature, under broad assumptions that certainly warrant the application of the qualitative conclusions to microwave circuits {1}. However, classic bifurcation theory is based on a *time-domain* description of the system being considered, which would be both impractical and inaccurate for microwave circuits.

A major contribution of this paper is to introduce a general algorithm for the detection of bifurcations of a parametrized microwave circuit, which is based on a *frequency-domain* description of the linear part of the network, and can thus take profit of advanced techniques for passive circuit modeling {3, 4}. The cornerstones are the piecewise harmonic-balance technique {5}, and a recently proposed approach to *local* stability analysis {6, 7}. To complete the global stability analysis, the algorithm is coupled to a standard continuation method {8}. The entire procedure is implemented in a general-purpose CAD environment, which means no limitations on the complexity of both the passive subnetwork and the active device models.

Global stability analysis has a vast potential for providing insight and control over some very important aspects of circuit behavior that have been dealt with so far by empirical methods only. We mention as examples the generation of spurious tones in microwave oscillators and the general problem of including stability requirements among the objectives of nonlinear circuit design. As a preliminary application the global stability analysis of a regenerative frequency divider is presented in this paper.

DESCRIPTION OF THE ALGORITHM

Let us consider a nonlinear microwave circuit continuously dependent on a parameter ρ. According to the piecewise harmonic-balance technique {5}, the electrical regime is described in terms of a state vector $\underline{X}$ whose elements are harmonics of the time-dependent state variables. The linear subnetwork is analyzed in the frequency domain. Periodic steady states are defined by the solutions of a nonlinear system of the form $\underline{E}(\underline{X}) = 0$, where the elements of $\underline{E}$ are harmonic-balance errors {5}. A perturbation analysis of the steady state {6, 7} leads to the characteristic equation $\Delta(s) = 0$ for the natural frequencies $s = \sigma + j\omega$. A general straightforward algorithm is available {7} for computing $\Delta(s)$, given an arbitrary network topology and a periodic steady state. The properties of the function $\Delta(s)$ are discussed in detail in {6}.

According to the definition, the existence of a bifurcation at $\rho = \rho_B$ requires the following set of mathematical conditions to be satisfied:

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$$\begin{cases} \underline{E}(\underline{X}, \rho_B) = 0 \\ \Delta(j\omega, \underline{X}, \rho_B) = 0 \end{cases} \quad (1)$$

$$\frac{d\sigma}{d\rho}(\rho_B) \neq 0 \quad (2)$$

where the dependence on ρ has been explicitly indicated. We first consider the bifurcations of periodic solutions of period $T_o = 2\pi/\omega_o$. A periodic steady state is symbolically denoted by ${}_kS^m$ {9}, where k is the number of unstable natural frequencies and m indicates a period mT_o (1 understood). Then the following fundamental types of bifurcations are possible {1, 9}:

1) D-type (double-point bifurcation)
   A simple real natural frequency crosses the origin at $\rho = \rho_B$, so that (1) are satisfied with $\omega = 0$. The exchange of stability is defined by

$$_kS + {}_{k\pm1}S \leftrightarrows {}_{k\pm1}S + {}_kS \; , \quad (3)$$

   where the states appearing first (second) on both sides of the arrows correspond to each other.

1s) Special case of D-type (regular turning point)
   Same as 1), but the creation or annihilation of two periodic states takes place at $\rho = \rho_B$. The exchange of stability is defined by

$$\phi \leftrightarrows {}_{k\pm1}S + {}_kS \; , \quad (4)$$

   where ϕ denotes the absence of solutions.

2) I-type (period-doubling bifurcation)
   Two simple complex-conjugate natural frequencies of the form $\sigma \pm j\omega_o/2$ cross the imaginary axis at $\rho = \rho_B$, so that (1) are satisfied with $\omega = \pm\, \omega_o/2$. The exchange of stability is defined by

$$_kS \leftrightarrows {}_{k\pm1}S + 2\,{}_kS^2 \; . \quad (5)$$

3) Hopf-type (spurious-exciting bifurcation)
   Two simple complex-conjugate natural frequencies cross the imaginary axis at $\rho = \rho_B$, so that (1) are satisfied with $0 < |\omega| < \omega_o/2$. The exchange of stability is defined by

$$_kS \leftrightarrows {}_{k\pm2}S + {}_k(\text{INVARIANT CLOSED CURVE}) \; , \quad (6)$$

   where the invariant closed curve represents a quasi-periodic regime which is stable for $k = 0$, unstable otherwise.

Since $\Delta(0)$ and $\Delta(j\omega_o/2)$ are real quantities {6}, (1) is always well conditioned from a mathematical viewpoint, that is, the number of real equations is equal to the number of real unknowns. This also explains why 1) - 3) represent the *fundamental* bifurcations: the existence of such bifurcations is mathematically possible in generic situations. On the other hand, more complex kinds of bifurcations requiring additional constraints to be imposed on the same variables appearing in (1) (e.g., $\omega = \omega_o/4$ for a period-quadrupling bifurcation) will only exist under exceptional circumstances.

To solve the system (1), $\underline{E}(\underline{X}, \rho) = 0$ is first solved for $\underline{X}(\rho)$ by a continuation method {10, 11}. Then D-type and I-type bifurcations are found by solving $\Delta(0) = 0$ and $\Delta(j\omega_o/2) = 0$ in the one-dimensional manifold $\underline{X}(\rho)$. Hopf bifurcations are found by solving $\Delta(j\omega) = 0$ in the two-dimensional manifold $[\omega, \underline{X}(\rho)]$. At each solution the condition (2) is easily checked by Nyquist analysis {6}.

The whole procedure is then repeated for the bifurcating branches. Since the stability of the circuit does not change, by definition, along a branch not containing bifurcations, a global stability picture for the circuit being considered is readily obtained in this way. Note that this implies that the stability of an *infinite* number of possible states becomes known by a *finite* number of operations.

As a final point, we shall briefly discuss the bifurcations of static solutions of the circuit equations. In this case the fundamental bifurcations are the D- and the Hopf-type {1, 2}. For microwave applications the latter plays an essential role in oscillator design and parasitic bias-circuit oscillations control in general microwave subsystems. The former may be of interest in relation with the design of DC-stable bias networks.

The conditions defining a bifurcation of a static solution are obviously much simpler than (1). If $\underline{X}_o$ is the DC (and the only nonzero) component of the state vector at the bifurcation, we must have

$$\begin{cases} \underline{E}_o(\underline{X}_o, \rho_B) = 0 \\ \det\left[\underline{1}_n - \underline{S}(\omega, \rho_B)\, \underline{S}_D(\omega, \underline{X}_o, \rho_B)\right] = 0 \end{cases} \quad (7)$$

$$\frac{d\sigma}{d\rho}(\rho_B) \neq 0 \quad (8)$$

where $\underline{S}$ is the conventional scattering matrix of the linear subnetwork (which may depend on the parameter ρ), and $\underline{S}_D$ is the small-signal scattering matrix of the nonlinear subnetwork describing its linearized behavior around the bias point defined by $\underline{X}_o$ ($\underline{1}_n$ represents an identity matrix of order equal to the number of subnetwork ports).

The exchange of stability at the bifurcation is defined by the following equations {1, 2}.

1) For the D-type bifurcation (assuming that a simple real eigenvalue changes sign):

$$ {}_kS^{\infty} + {}_{k\pm1}S^{\infty} \leftrightarrows {}_{k\pm1}S^{\infty} + {}_kS^{\infty} \quad (9) $$

or

$$ {}_kS^{\infty} \leftrightarrows {}_{k\pm1}S^{\infty} + 2\,{}_kS^{\infty} \;. \quad (10) $$

1s) For the regular turning point

$$ \phi \leftrightarrows {}_{k\pm1}S^{\infty} + {}_kS^{\infty} \;. \quad (11) $$

2) For the Hopf bifurcation:

$$ {}_kS^{\infty} \leftrightarrows {}_{k\pm2}S^{\infty} + 2\,{}_kS \;. \quad (12) $$

In (9) - (12) the superscript ∞ denotes a DC state.

In all cases, the number of equations in (7) equals the number of real unknowns, so that the system is generally solvable from the mathematical viewpoint. The solution is now simplified by the fact that the second of eqs. (7) simply states that one of the eigenvalues (in a conventional sense) of the matrix $\underline{S}\ \underline{S}_D$ must be equal to 1 at the bifurcation. Thus a convenient way of solving (7) is now as follows: i) the first of (7) is solved for $\underline{X}_o(\rho)$ by a continuation method; ii) to find D-type bifurcations, the one-dimensional manifold $\underline{X}_o(\rho)$ is searched for the points ρ_B at which one eigenvalue of $\underline{S}\ \underline{S}_D$ becomes unity: iii) to find Hopf-type bifurcations the two-dimensional manifold

$[\omega\ ,\ \underline{X}_o(\rho)]$ is searched for those points $(\omega\ ,\ \rho_B)$

at which one eigenvalue of $\underline{S}\ \underline{S}_D$ becomes unity. To verify (8) we only have to check that the magnitude of the above mentioned eigenvalue is < 1 at $\rho_B - \delta\rho$ and > 1 at $\rho_B + \delta\rho\,(\delta\rho << \rho_B)$, or conversely.

GLOBAL STABILITY ANALYSIS OF AN ACTIVE FREQUENCY DIVIDER

A circuit topology similar to the one described in {12} was adopted; the details of the device model are shown in fig. 1.

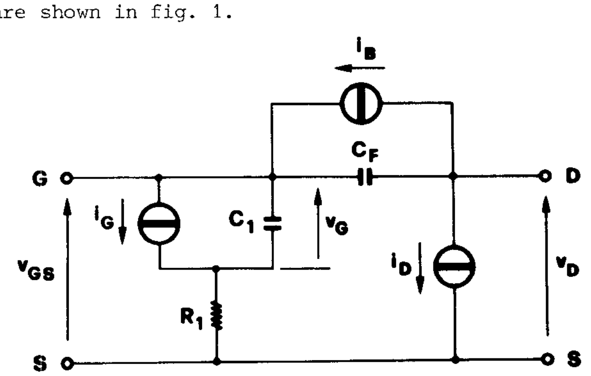

Fig. 1 - FET model used in the simulation (I_{DSS} = 40mA, C_{10} = 0.42pF, V_p = -1.91V) All elements shown are nonlinear.

This circuit was first designed by a general-purpose nonlinear optimization program {13} for input and output frequencies $f_{IN} = 10$ GHz, $f_{OUT} = 5$ GHz, and for the following set of specifications:

- available input power at 10 GHz = 6 mW
- conversion gain > 0 dB
- spectral purity of 5 GHz output signal > 16 dB
- input return loss at 10 GHz > 10 dB.

4 harmonics of the output frequency were used in the design.

A global stability analysis of the circuit thus obtained was then carried out. A one-dimensional parametrization with $\rho = P_{IN}$ (available power of the 10 GHz pump expressed in mW) was adopted. The results of this analysis are presented in fig. 2, where the quantity

$$ M = (||\underline{X}|| - ||\underline{X}_o||)^{1/2} \quad (13) $$

is plotted against $\rho = P_{IN}$ ($||\ \ ||$ denotes the norm). The bias point $\underline{X}_o$ was kept fixed throughout the calculation. The range of interest is $0 \leq \rho \leq 10$.

The "multiplier branch" (fig. 2) was first determined by a continuation method starting from $\rho = 0$. A local stability analysis of the bias point chosen (V_{GO} = -1.9V, V_{DO} = 6V) revealed that the circuit is DC stable; thus the multiplier branch is stable in the neighborhood of the origin. Only one solution of the system (1) was found on the multiplier branch within the range of interest: an I-type bifurcation at $\rho = \rho_I \simeq 5.1$ (point I). Thus the multiplier branch is stable for $\rho < \rho_I$ and unstable for $\rho > \rho_I$.

Starting at $\rho = \rho_I$, the "divider branch" was then determined by a continuation method. Since this branch starts with $\rho < \rho_I$, the bifurcation at point I is *subcritical* {1}, i.e., is described by (5) with the arrow pointing from right to left, k = 1 and the "minus" sign of the right-hand side. Thus the divider branch is unstable in the vicinity of the bifurcation. Only one solution of the system (1) was found on the divider branch within the range of interest: a D-type bifurcation corresponding to the regular turning point D. At point D this branch becomes stable, which can be established by performing a local stability analysis anywhere beyond the turning point (this analysis is required to show that the bifurcation may be described by (4) with k = 0). In particular, the nominal operating point A is found to be stable. The divider branch is unstable with one positive real natural frequency between I and D. The unstable branch I D determines the existence of a hysteresis cycle around threshold.

Note that each point of the divider branch is actually representative of two states (e.g., A and $\bar{A}$ in fig. 2) only differing in the sign of the odd

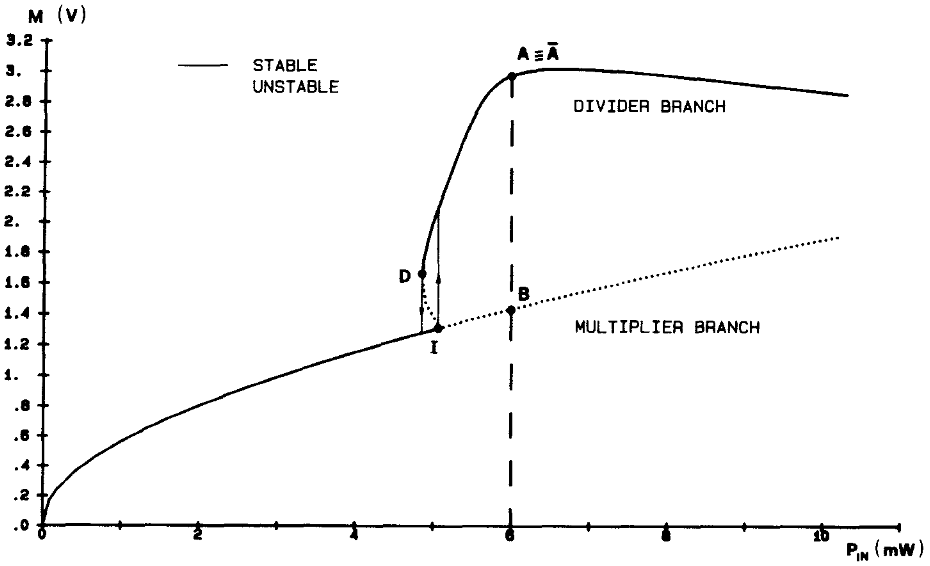

Fig. 2 - Bifurcation diagram of a regenerative frequency divider

harmonics, and thus associated with the same value of M.

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